

§ 2.5: Quadratic Functions; maxima and Minima

Graphing Quadratic Functions Using the Standard Form

A **quadratic function** is a function f of the form

$$f(x) = ax^2 + bx + c$$

where a, b , and c are real numbers and $a \neq 0$.

For example, if we take $a = 1, b = c = 0$, we get the simple quadratic function

$$f(x) = x^2.$$

The graph of any quadratic function is called a **parabola**, and can be obtained from the graph of $f(x) = x^2$ by the transformations discussed in § 2.4.

Standard Form of a Quadratic Function

A quadratic function $f(x) = ax^2 + bx + c$ can be expressed in the **standard form**

$$f(x) = a(x - h)^2 + k$$

by completing the square. The graph of f is a parabola with **vertex** (h, k) ; the parabola opens upward if $a > 0$ or downward if $a < 0$.

Example 1	Standard Form of a Quadratic Function
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Let $f(x) = 3x^2 - 12x + 20$.

(a) Express f in standard form.

(b) Sketch the graph of f .

Maximum and Minimum Values of Quadratic Functions

Maximum or Minimum Value of a Quadratic Function

Let f be a quadratic function with standard form $f(x) = a(x - h)^2 + k$. The maximum or minimum value of f occurs at $x = h$.

If $a > 0$, then the **minimum value** of f is $f(h) = k$.

If $a < 0$, then the **maximum value** of f is $f(h) = k$.

Example 2	Minimum Value of a Quadratic Function
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Consider the quadratic function $f(x) = 5x^2 + 20x + 19$.

- (a) Express f in standard form.
- (b) Find the minimum value of f .

Example 3	Maximum Value of a Quadratic Function
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Consider the quadratic function $f(x) = -2x^2 + 14x - 10$.

- (a) express f in standard form.
- (b) Find the maximum value of f .

We now derive a formula for the maximum or minimum of the quadratic function $f(x) = ax^2 + bx + c$ in terms of a , b , and c .

$$\begin{aligned} f(x) &= ax^2 + bx + c \\ &= a\left(x^2 + \frac{b}{a}x\right) + c && \text{Factor } a \text{ from the } x\text{-terms} \\ &= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - a\left(\frac{b^2}{4a^2}\right) && \text{Complete the square.} \\ &= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} && \text{Factor and simplify} \end{aligned}$$

Thus we get

Maximum of Minimum Value of a Quadratic Function

The maximum or minimum value of a quadratic function $f(x) = ax^2 + bx + c$ occurs at

$$x = -\frac{b}{2a},$$

the maximum or minimum value is

$$f\left(-\frac{b}{2a}\right) = c - \frac{b^2}{4a}$$

Example 4	Finding Maximum and Minimum Values of Quadratic Functions
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Find the maximum or minimum value of each quadratic function.

(a) $f(x) = x^2 + 4x$

(b) $f(x) = -2x^2 + 4x - 5$

Example 5	Advertising
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The effectiveness of a television commercial depends on how many times a viewer watches it. After some experiments and advertising agency found that if the effectiveness E is measured on a scale of 1 to 10, then

$$E(n) = \frac{2}{3}n - \frac{1}{90}n^2$$

where n is the number of times a viewer watches a given commercial. For a commercial to have maximum effectiveness, how many times should a viewer watch it?

Example 6	Domain and Range of Quadratic Functions
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Find the domain and range of each of the following quadratic functions.

(a) $f(x) = -x^2 + 4x - 3$

(b) $f(x) = 2x^2 + 6x - 7$

Homework

Due: _____

6 – 44 (even)